10. Hierarchical Modeling

Reading

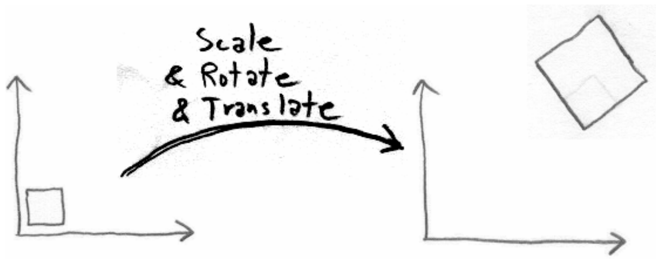
- Angel, sections 9.1 - 9.6
  [reader pp. 169-185]
- *OpenGL Programming Guide*, chapter 3
  - Focus especially on section titled "Modelling Transformations".

Symbols and instances

Most graphics APIs support a few geometric **primitives**:

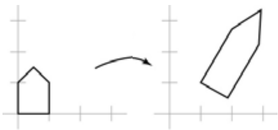
- spheres
- cubes
- triangles

These symbols are **instanced** using an **instance transformation**.

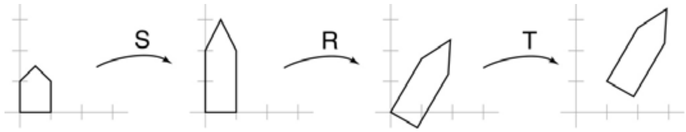


Use a series of transformations

Ultimately, a particular geometric instance is transformed by one combined transformation matrix:



But it's convenient to build this single matrix from a series of simpler transformations:

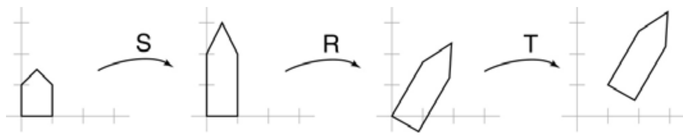


We have to be careful about how we think about composing these transformations.

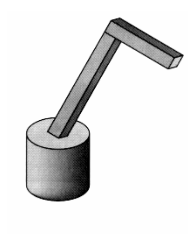
(Mathematical reason: Transformation matrices don't commute under matrix multiplication)

Two ways to compose xforms

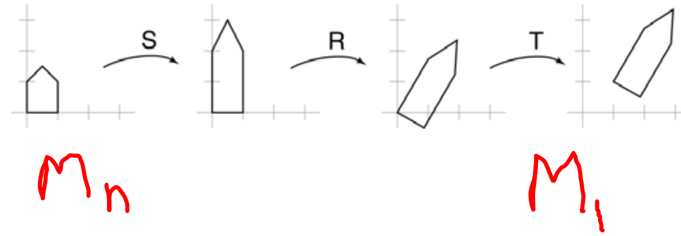
Method #1:
Express every transformation with respect to global coordinate system:



Method #2:
Express every transformation with respect to a “parent” coordinate system created by earlier transformations:



#1: Xform for global coordinates



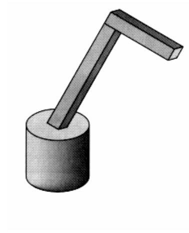
FinalPosition = M_1 * M_2 * … * M_n * InitialPosition

Apply First

Apply Last

Note: Positions are column vectors: $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

#2: Xform for coordinate system

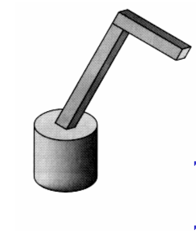


$$\text{FinalPosition} = M_1 * M_2 * \ldots * M_n * \text{InitialPosition}$$

Apply First

Apply Last

Xform direction for coord. sys



Global coord sys

Coord sys resulting from M1.

Coord sys resulting from M * M2

Local coord sys, resulting from M1 * M2 * … * Mn

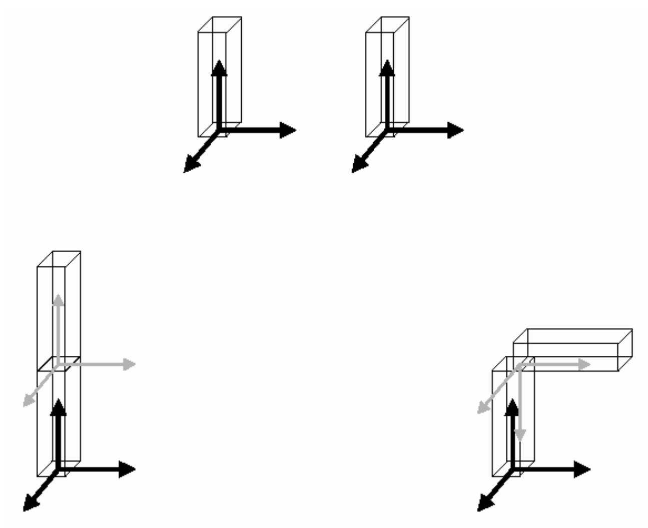
$$\text{FinalPosition} = M_1 * M_2 * \ldots * M_n * \text{InitialPosition}$$

Translate/Rotate:

FROM previous coord sys
TO new one

with transformation expressed in the ‘previous’ coordinate system.

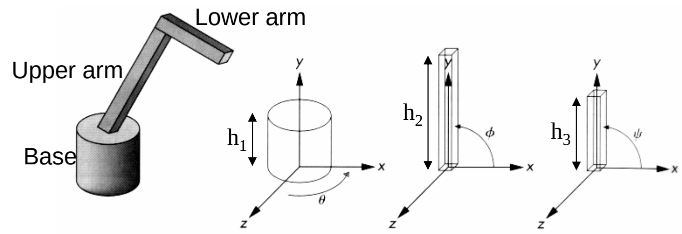
Connecting primitives



3D Example: A robot arm

Consider this robot arm with 3 degrees of freedom:

- Base rotates about its vertical axis by θ
- Upper arm rotates in its *xy*-plane by ϕ
- Lower arm rotates in its *xy*-plane by ψ



Q: What matrix do we use to transform the base?

Q: What matrix for the upper arm?

Q: What matrix for the lower arm?

Robot arm implementation

The robot arm can be displayed by keeping a global matrix and computing it at each step:

```
Matrix M_model;

main()
{
    . . .
    robot_arm();
    . . .
}

robot_arm()
{
    M_model = R_y(theta);
    base();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model = R_y(theta)*T(0,h1,0)*R_z(phi)
                      *T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Do the matrix computations seem wasteful?

Robot arm implementation, better

Instead of recalculating the global matrix each time, we can just update it *in place* by concatenating matrices on the right:

```
Matrix M_model;

main()
{
    . . .
    M_model = Identity();
    robot_arm();
    . . .
}

robot_arm()
{
    M_model *= R_y(theta);
    base();
    M_model *= T(0,h1,0)*R_z(phi);
    upper_arm();
    M_model *= T(0,h2,0)*R_z(psi);
    lower_arm();
}
```

Robot arm implementation, OpenGL

OpenGL maintains a global state matrix called the **model-view matrix**, which is updated by concatenating matrices on the ***right***.

```
main()
{
    . . .
    glMatrixMode( GL_MODELVIEW );
    glLoadIdentity();
    robot_arm();
    . . .
}

robot_arm()
{
    glRotatef( theta, 0.0, 1.0, 0.0 );
    base();
    glTranslatef( 0.0, h1, 0.0 );
    glRotatef( phi, 0.0, 0.0, 1.0 );
    lower_arm();
    glTranslatef( 0.0, h2, 0.0 );
    glRotatef( psi, 0.0, 0.0, 1.0 );
    upper_arm();
}
```

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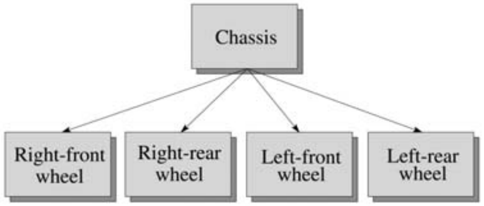
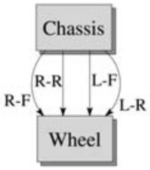
Hierarchical modeling

Hierarchical models can be composed of instances using trees or DAGs:





- edges contain geometric transformations
- nodes contain geometry (and possibly drawing attributes)

How might we draw the tree for the robot arm?

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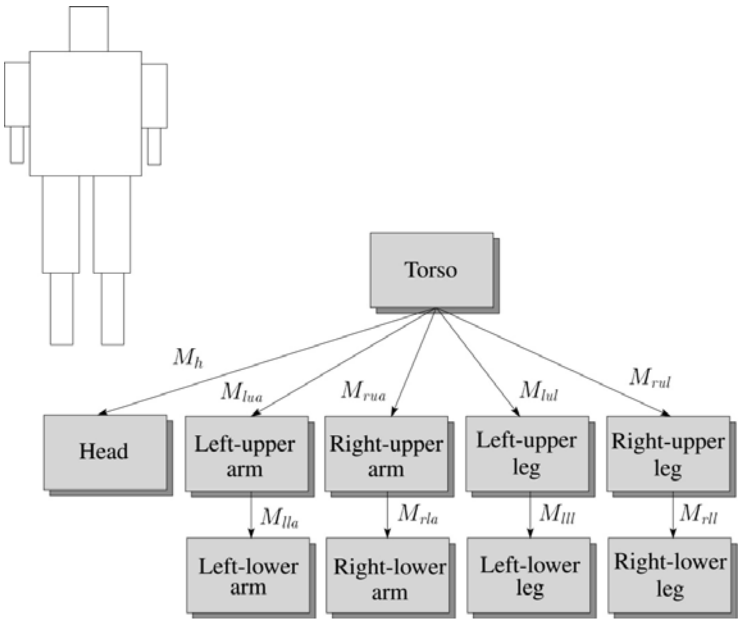
A complex example: human figure



Q: What's the most sensible way to traverse this tree?

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Human figure implementation, OpenGL

```
figure()
{
    torso();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        head();
    glPopMatrix();
    glPushMatrix();
        glTranslate( ... );
        glRotate( ... );
        left_upper_arm();
        glPushMatrix();
            glTranslate( ... );
            glRotate( ... );
            left_lower_arm();
        glPopMatrix();
     glPopMatrix();
     . . .
}
```

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Animation

The above examples are called **articulated models**:

- rigid parts
- connected by joints

They can be animated by specifying the joint angles (or other display parameters) as functions of time.

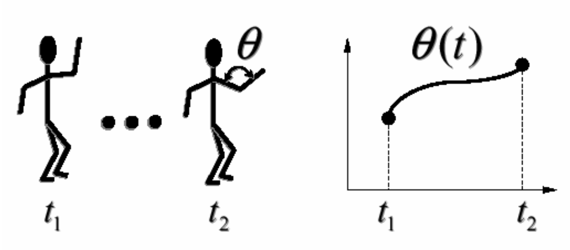
Key-frame animation

The most common method for character animation in production is **key-frame animation**.

- Each joint specified at various **key frames** (not necessarily the same as other joints)
- System does interpolation or **in-betweening**

Doing this well requires:

- A way of smoothly interpolating key frames: **splines**
- A good interactive system
- A lot of skill on the part of the animator



Scene graphs

The idea of hierarchical modeling can be extended to an entire scene, encompassing:

- many different objects
- lights
- camera position

This is called a **scene tree** or **scene graph**.