6. Ray Tracing

Reading

Required:

- Watt, sections 1.3-1.4, 12.1-12.5.1.
- T. Whitted. An improved illumination model for shaded display. Communications of the ACM 23(6), 343-349, 1980. [In the reader.]

Further reading:

- A. Glassner. An Introduction to Ray Tracing. Academic Press, 1989. [In the lab.]
- K. Turkowski, "Properties of Surface Normal Transformations," Graphics Gems, 1990, pp. 539-547. [In the reader.]

Geometric optics

Modern theories of light treat it as both a wave and a particle.

We will take a combined and somewhat simpler view of light – the view of **geometric optics**.

Here are the rules of geometric optics:

- Light is a flow of photons with wavelengths. We'll call these flows "light rays."
- Light rays travel in straight lines in free space.
- Light rays do not interfere with each other as they cross.
- Light rays obey the laws of reflection and refraction.
- Light rays travel form the light sources to the eye, but the physics is invariant under path reversal (reciprocity).

Synthetic pinhole camera

The most common imaging model in graphics is the synthetic pinhole camera: light rays are collected through an infinitesimally small hole and recorded on an **image plane**.
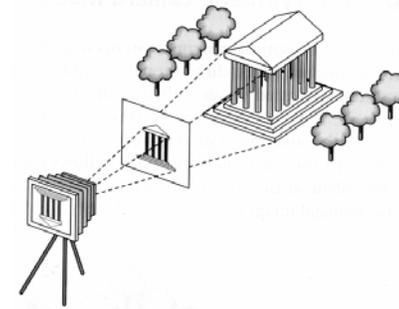
For convenience, the image plane is usually placed in front of the camera, giving a non-inverted 2D projection (image).

Viewing rays emanate from the **center of projection** (COP) at the center of the lens (or pinhole).
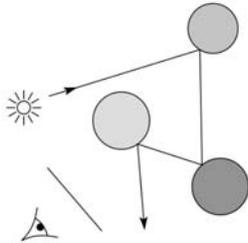
The image of an object point P is at the intersection of the viewing ray through P and the image plane.
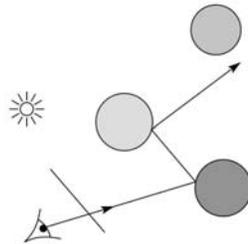
Eye vs. light ray tracing

Where does light begin?

At the light: light ray tracing (a.k.a., forward ray tracing or photon tracing)

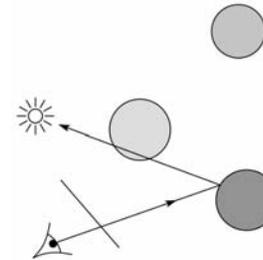At the eye: eye ray tracing (a.k.a., backward ray tracing)

We will generally follow rays from the eye into the scene.

Precursors to ray tracing
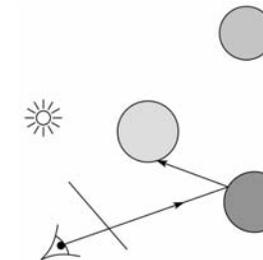
Local illumination

 • Cast one eye ray, then shade according to light
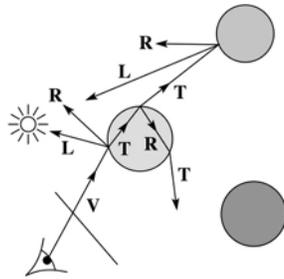
Appel (1968)

 • Cast one eye ray + one ray to light

Whitted ray-tracing algorithm

In 1980, Turner Whitted introduced ray tracing to the graphics community.

- Combines eye ray tracing + rays to light
- Recursively traces rays

Algorithm:

1. For each pixel, trace a **primary ray** in direction **V** to the first visible surface.

2. For each intersection, trace **secondary rays**:

- **Shadow rays** in directions L_i to light sources
- **Reflected ray** in direction **R**.
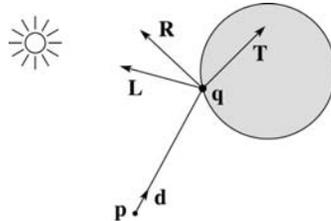- **Refracted ray** or **transmitted ray** in direction **T**.

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Whitted algorithm (cont'd)

Let's look at this in stages:

Primary rays

Shadow rays

Reflection rays

Refracted rays

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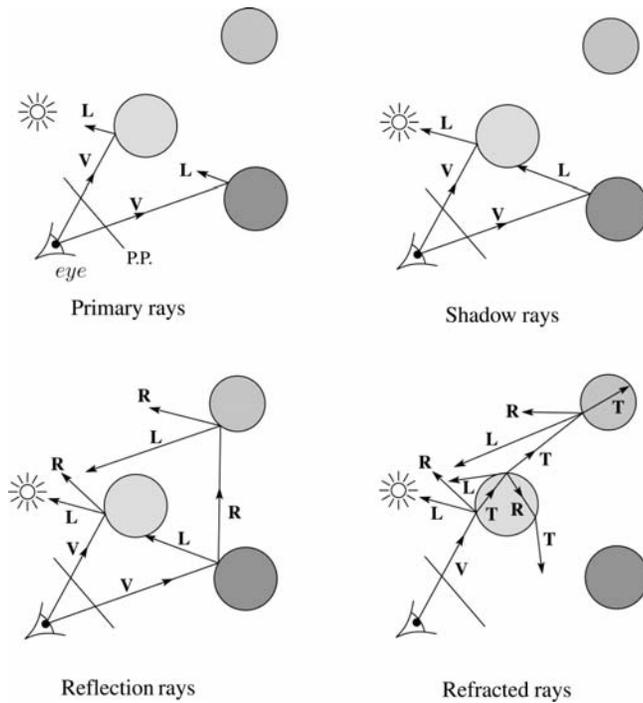
Shading

A ray is defined by an origin **P** and a unit direction **d** and is parameterized by t:

$$P + t\mathbf{d}$$

Let $I(P, \mathbf{d})$ be the intensity seen along that ray. Then:

$$I(P, \mathbf{d}) = I_{direct} + I_{reflected} + I_{transmitted}$$

where

- I_{direct} is computed from the Phong model
- $I_{reflected} = k_r I(Q, \mathbf{R})$
- $I_{transmitted} = k_t I(Q, \mathbf{T})$

Typically, we set $k_r = k_s$ and $k_t = 1 - k_s$.

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Reflection and transmission
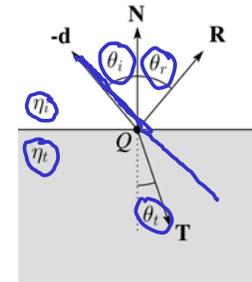
Law of reflection:

$$\theta_i = \theta_r$$

Snell's law of refraction:

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$

where η_i , η_t are **indices of refraction**.

$$\theta_t = \sin^{-1}\left(\frac{\eta_i \sin\theta_i}{\eta_t}\right)$$

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Total Internal Reflection

The equation for the angle of refraction can be computed from Snell's law:
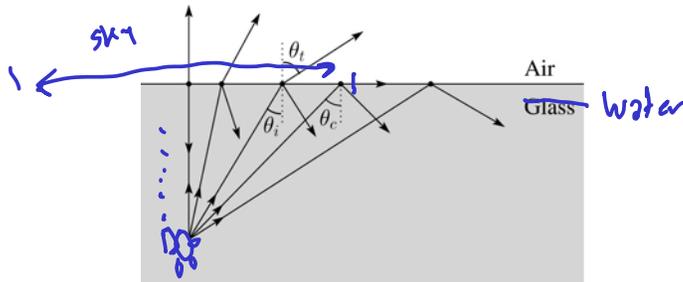
What happens when $\eta_i > \eta_t$?

When θ_t is exactly 90°, we say that θ_I has achieved the "critical angle" θ_c.

For $\theta_I > \theta_c$, *no rays are transmitted*, and only reflection occurs, a phenomenon known as "total internal reflection" or TIR.

Error in Watt!!

In order to compute the refracted direction, it is useful to compute the cosine of the angle of refraction in terms of the incident angle and the ratio of the indices of refraction.

On page 24 of Watt, he develops a formula for computing this cosine. Notationally, he uses μ instead of η for the index of refraction in the text, but uses η in Figure 1.16(!?), and the angle of incidence is ϕ and the angle of refraction is θ.

Unfortunately, he makes a grave error in computing $\cos\theta$. He also has some errors in the figures on the same page.
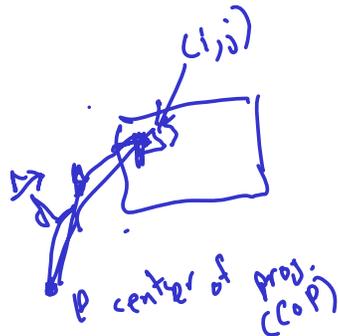
Consult the errata for important corrections!

Ray-tracing pseudocode

We build a ray traced image by casting rays through each of the pixels.

function *traceImage* (scene):
   for each pixel (i,j) in image
      S = *pixelToWorld*(i,j)
      P = **COP**
      $\mathbf{d}$ = $(S - P)/\| S - P \|$
      $I(i,j)$ = *traceRay*(scene, P, $\mathbf{d}$)
   end for
end function

(i,j)

P center of proj.
(COP)

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$x = P + t\vec{d}$

Ray-tracing pseudocode, cont'd

if depth ≥ 3 return 0

function *traceRay*(scene, P, $\mathbf{d}$): depth)
   (t, **N**, mtrl) $\leftarrow$ scene.*intersect* (P, $\mathbf{d}$)
   $Q \leftarrow$ ray (P, $\mathbf{d}$) evaluated at t
   $Q = P + t\hat{d}$
   I = *shade*(**q**, **N**, mtrl, scene)
   R = *reflectDirection*(**N**, -**d**)
   I $\leftarrow$ I + mtrl.k_r ∗ *traceRay*(scene, Q, **R**)
   if ray is entering object **then**
      n_i = index_of_air
      n_t = mtrl.index
   else
      n_i = mtrl.index
      n_t = index_of_air
   if (mtrl.k_t > 0 **and** *notTIR* (n_i, n_t, **N**, -**d**)) **then**
      T = *refractDirection* (n_i, n_t, **N**, -**d**)
      I $\leftarrow$ I + mtrl.k_t ∗ *traceRay*(scene, Q, **T**)
   end if
   return I
end function

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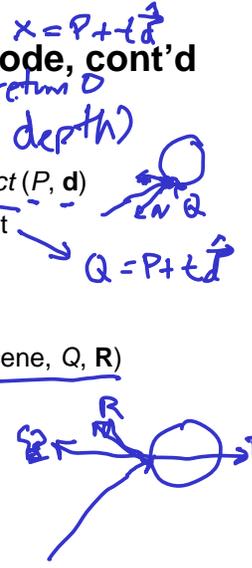
Terminating recursion

Q: How do you bottom out of recursive ray tracing?

Possibilities:

Shading pseudocode

Next, we need to calculate the color returned by the *shade* function.

function *shade*(mtrl, scene, Q, **N**, **d**):

    I ← mtrl.k_e + mtrl. k_a * scene->I_a

    for each light source ℓ **do**:

        atten = ℓ -> *distanceAttenuation*(Q) *

            ℓ -> *shadowAttenuation*(scene, Q)

        I ← I + atten*(diffuse term + spec term)

    end for

    return I

end function

Shadow attenuation

Computing a shadow can be as simple as checking to see if a ray makes it to the light source.
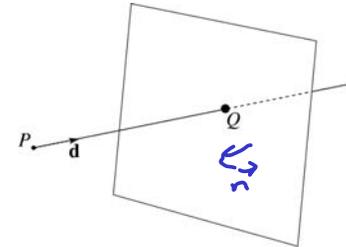
For a point light source:

function *PointLight***::***shadowAttenuation(*scene*, P)*

    d = (ℓ.position - *P***).***normalize*()

    (t, **N**, mtrl) ← scene.*intersect*(*P*, **d**)

    Q ← ray(t)

    if *Q* is before the light source **then**:

        atten = 0

    else

        atten = 1

    end if

    return atten

end function

Q: What if there are transparent objects along a path to the light source?

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Ray-plane intersection

We can write the equation of a plane as:

$$ax + by + cz + d = 0$$

$$\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The coefficients *a*, *b*, and *c* form a vector that is normal to the plane, **n** = [*a b c*]^{T}. Thus, we can re-write the plane equation as:

$$\vec{n} \cdot \vec{x} + d = 0$$

We can solve for the intersection parameter (and thus the point):
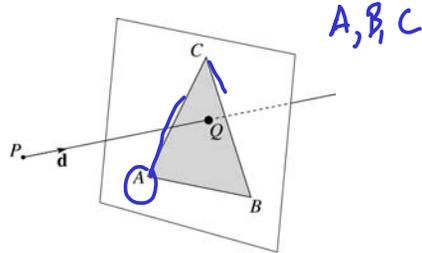
$$\vec{x} = P + t\vec{d}$$

$$\vec{n} \cdot (P + t\vec{d}) + d = 0$$

$$\vec{n} \cdot P + \vec{n} \cdot (t\vec{d}) + d = 0$$

$$t = \dots$$

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Ray-triangle intersection

A,B C

To intersect with a triangle, we first solve for the equation of its supporting plane:

$$\vec{n} = \vec{AC} \times \vec{AB}$$
$$(C-A) \times (B-A)$$

$$d =$$

Then, we need to decide if the point is inside or outside of the triangle.

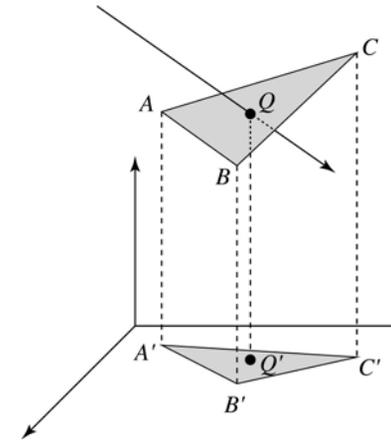Solution 1: compute barycentric coordinates from 3D points.

α, β, δ

What do you do with the barycentric coordinates?

$\delta > 1 \qquad \beta > 1 \qquad \delta > 1$

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Ray-triangle intersection

Solution 2: project down a dimension and compute barycentric coordinates from 2D points.

Why is solution 2 possible? Why is it legal? Why is it desirable? Which axis should you "project away"?

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Interpolating vertex properties

The barycentric coordinates can also be used to interpolate vertex properties such as:

- material properties
- texture coordinates
- normals

For example:

$$k_d(Q) = \alpha k_d(A) + \beta k_d(B) + \gamma k_d(C)$$

Interpolating normals, known as Phong interpolation, gives triangle meshes a smooth shading appearance. (Note: don't forget to normalize interpolated normals.)

Epsilons

Due to finite precision arithmetic, we do not always get the exact intersection at a surface.

Q: What kinds of problems might this cause?

Q: How might we resolve this?

Intersecting with xformed geometry

In general, objects will be placed using transformations.   What if the object being intersected were transformed by a matrix M?

Apply M^{-1} to the ray first and intersect in object (local) coordinates!

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Intersecting with xformed geometry

The intersected normal is in object (local) coordinates. How do we transform it to world coordinates?

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