Computer Graphics	Prof. Bill Mark
CS 384g	Fall 2004

Homework #2

Textures, shading, parametric curves, and parametric surfaces

Assigned: Tuesday, November 2, 2004

**Due: Tuesday, November 9, 2004**
(at the beginning of class)

Directions: Please provide short written answers to the following questions. Feel free to talk over the problems in general terms with classmates, but please *answer the questions on your own*.

Name:_____

1. Textures and shading

Texture mapping is a powerful method for modulating surface appearance with functions mapped over a surface. As always, when rendering such a surface, aliasing artifacts will arise and can be reduced by integrating the light rays over the area of a pixel. Alternatively, to reduce the aliasing specifically due to textures, the texture images themselves can be pre-filtered and used to accelerate the integration process.

In this problem, we consider the simple case of a plane viewed head-on with parallel viewing rays, illuminated by a single directional light source:
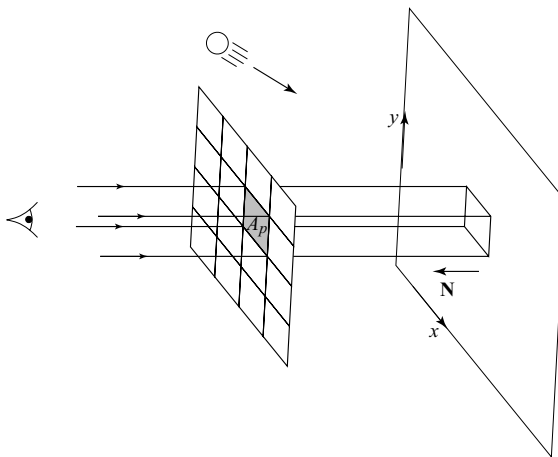
and shaded using the local, Phong lighting model (no emissive, no ambient, no interreflections):

$$I_{phong} = I_\ell k_d \left(\mathbf{N} \cdot \mathbf{L}\right)_+ + I_\ell k_s \left(\mathbf{V} \cdot \mathbf{R}\right)_+^{n_s}$$

We'll assume that $\mathbf{N} = \mathbf{V} = (0,0,1)^T$ and $\mathbf{L} = (-\sin 30^\circ, 0, \cos 30^\circ)^T$. For simplicity, we'll also assume that all values are grayscale (just scalar-valued intensities and coefficients). We can anti-alias with a box filter by integrating the light over the area of a pixel:

$$I(p_x, p_y) = \frac{1}{A_p} \int_{A_p} I_{phong}(x, y) \, dx\, dy$$

Alternatively, we can use pre-filtering and simply compute:

$$\tilde{I}(p_x, p_y) = \tilde{I}_{phong}(p_x, p_y)$$

where, in this latter case, one of the material parameters is replaced with the pre-filtered value before computing the Phong shading. For this problem, you may assume that all material parameters are constant, except for the one explicitly being varied.

a. Let's say the surface is modulated by a diffuse texture, $k_d(x,y)$. The pre-filtering process allows us to quickly evaluate:

$$\tilde{k}_d = \frac{1}{A_p} \int_{A_p} k_d(x, y) \, dx\, dy$$

using lookup tables, which we can then use to compute $\tilde{I}(p_x, p_y)$. Do we expect to find

$I(p_x, p_y) = \tilde{I}(p_x, p_y)$? Justify your answer. You may assume that the pre-integration with look-up tables is exact for the purposes of this problem.

b. If the texture instead modulates $n_s(x,y)$ and is pre-filtered as in (a), will $I(p_x, p_y) = \tilde{I}(p_x, p_y)$?

c. Now set $k_s = 0$ and bump map the surface with normals derived from a displacement map $d(x, y) = \cos(x)$. If we compute the "normal map" $\mathbf{N}(x, y)$ and then pre-filter that map as above, will $I(p_x, p_y) = \tilde{I}(p_x, p_y)$?

d. Keeping the same setup as in (c), what happens if we now change the lighting so that $\mathbf{L} = (-\sin 60^\circ, 0, \cos 60^\circ)^T$ and pre-filter in the same way. Will $I(p_x, p_y) = \tilde{I}(p_x, p_y)$?

2. **Parametric vs. geometric continuity**

In this problem, we examine the differences between parametric and geometric continuity of curves. Recall that C^1 continuity implies that the curve has no sudden jumps in it for neighboring parametric values (i.e., is C^0 continuous) and is also continuous in the 1st parametric derivative. G^1 continuity means that all points on the curve have immediate neighbors (i.e., the curve is G^0), and the direction of the tangent to the curve is continuous. Note also that the tangent to a curve points in the same direction as the first parametric derivative.

Now consider the following control polygon defined by consecutive sets of Bezier control points (V_0, V_1, V_2, V_3) and (W_0, W_1, W_2, W_3):
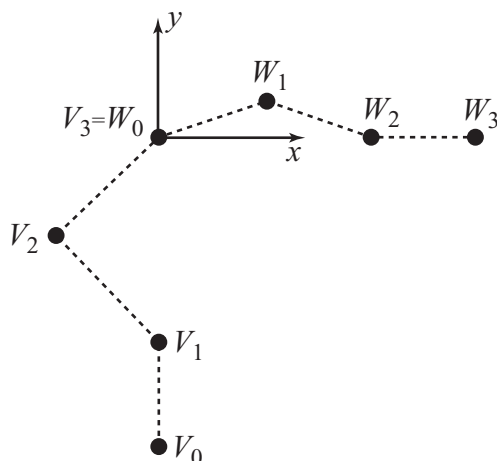

where:

$$V_0 = \begin{bmatrix} 0 \\ -3 \end{bmatrix} \qquad V_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \qquad V_2 = V_3 - a\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad V_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad W_1 = W_0 + b\begin{bmatrix} 1 \\ c \end{bmatrix} \qquad W_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad W_3 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

a. Sketch the control points and curve you get when setting $a = b = c = 1$. Note, your sketch does not need to be precise, but it should show which control points are interpolated, suggest the directions of the tangents at those points, and fit within the regions to which the curve should be restricted. (This note applies to the remainder of the problem.)

b. Sketch the control points and curve you get when setting $a = 1$, $b = 1/2$, $c = 1$.

c. Sketch the control points and curve you get when setting $a = b = 1$, $c = 0$.

d. For $a, b > 0$, what constraints on a, b, and/or c will ensure that the curve is C^1 continuous?

e. For $a, b > 0$, what constraints on a, b, and/or c will ensure that the curve is G^1 continuous?

f. Now consider the case where $a = b = 0$. Sketch the component curves $x(u)$ vs. u and $y(u)$ vs. u as two separate plots where u varies from -1 to 0 for the first set of Bezier control points, (V_0, V_1, V_2, V_3), and then 0 to 1 for the second set, (W_0, W_1, W_2, W_3).

g. Sketch the control points and curve you get when setting $a = b = 0$ (as in (f)).

h. Is the curve in part (f) C^1 continuous, G^1 continuous, both, or neither? Justify your answer.

3. Cubic Bezier tensor product patches

Below is a diagram indicating the sixteen control points for specifying a cubic Bezier tensor product patch, $S(u,v)$.
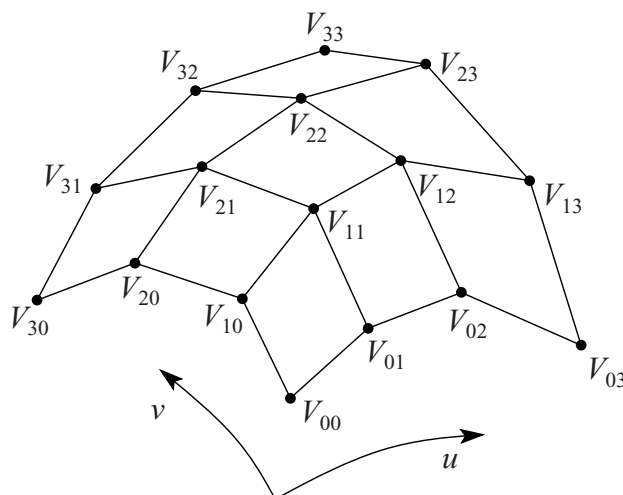
Answer each of the following questions, and justify your answers.

a. Which control points are guaranteed to be interpolated by the surface?

b. The tangent plane to a point on the surface is determined by that point and any two linearly independent vectors which are tangent to the surface at that point and thus span the plane. Two such tangent vectors are $\partial S / \partial u$ and $\partial S / \partial v$ evaluated at the point of interest. Which control points determine the tangent plane to the surface at $u = v = 0$?

c. Which control points determine the normal to the surface at $u = v = 1/2$?

d. Will the surface lie within the convex hull of its control points?

e. Consider the case where the control points are co-planar. How would you characterize the shape of the surface? Will all of the control points in general be interpolated if they are coplanar?

4. Final project

Send email to both Chris and myself describing who you plan to work with and what you plan to develop for your final project. Explain your division of labor, and describe the artifact you hope to produce. This email is due by midnight of Thursday November 11. Later in that week, we will set up a meeting with each team to discuss your project. The project itself will be demoed in class on Tuesday November 30, with a concise web page write-up (with artifacts) due at 11:59pm on Friday Dec 3.