Computer Graphics	**Prof. William Mark**
CS 384g	**Fall 2004**

Homework #1

Sampling theory, image processing, affine transformations

Assigned: Sept 16, 2004
Due: Sept 28, 2004
(at the beginning of class)

Directions: Please provide short written answers to the following questions. Feel free to talk over the problems in general terms with classmates, but please *answer the questions on your own*.

Name:____

1. Fourier transforms and signal reconstruction
40 Points

In this problem, you will take a closer look at convolution, Fourier transforms and signal reconstruction. We begin with a couple of preliminaries.

i) Recall that the box function II(*x*) is defined as:

$$\mathrm{II}(x) = \begin{cases} 1 & |x| < 1/2 \\ 1/2 & |x| = 1/2 \\ 0 & |x| > 1/2 \end{cases}$$

and its Fourier transform is sinc(*s*) = sin(*πs*)/*πs*. Likewise, the Fourier transform of sinc(*x*) is II(*s*). More generally, for some constant frequency ω_0, the Fourier transform of $\mathrm{II}(\omega_0 x)$ is $\frac{1}{\omega_0}\mathrm{sinc}(\frac{1}{\omega_0}s)$. Also, we can show that sinc(0) = 1. This transform pair is depicted graphically in the lecture notes.

ii) The dirac delta function δ (x) can be defined as:

δ (x) = 0, for all x $\neq 0$

δ (0) = ∞

and with the property that $\int_{-\infty}^{\infty} \delta(x)dx = 1$

The Fourier transform of

$$\cos\left(2\pi f_0 x\right)$$

is

$$\frac{1}{2}\delta(x - f_0) + \frac{1}{2}\delta(x + f_0),$$

and vice-versa. The constant f_0 represents the frequency of the cosine function. This transform pair is also depicted graphically in the lecture notes.

iii) If the Fourier transform of h(x) is H(s), then the Fourier transform of $\mathrm{k} \cdot \mathrm{h(x)}$ is $\mathrm{k} \cdot \mathrm{H(s)}$.

iv) The hat function, Λ(*x*), is defined as:

$$\Lambda(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

We sample a function by multiplying with the comb or shah function:

$$\hat{f}(x) = f(x)\mathrm{III}(x) = \sum_{i=-\infty}^{i=\infty} f(i)\delta(x - i)$$

We reconstruct by convolving with a reconstruction filter *r*(*x*):

$$\tilde{f}(x) = r(x) * \hat{f}(x) = \sum_{i=-\infty}^{i=\infty} f(i)r(x - i)$$

Problem 1 (cont'd.)

Now we get to the problems you need to solve. (Hint: There is an easy way and a hard way to do these problems. The easy way is to use various simple properties of the Fourier transform.)

a) **Convolution:**

Given a function $f(x) = \cos(2\pi x) + \cos(8\pi x)$.

i) What is its Fourier transform (in equation form)? Plot the magnitude of the Fourier transform, with properly labeled axes.

ii) Given a convolution filter h(x) = sinc(3x), what is the function that results from convolving f(x) with h(x)?

b) **Sampling and Reconstruction:**

Given the same function $f(x) = \cos(2\pi x) + \cos(8\pi x)$.

i) Plot this function.

ii) If we sample this signal at points 0, 1/8, 2/8, 3/8, …, and then reconstruct it with a linear reconstruction kernel, h(x) = Λ(8x), what is the plot of the reconstructed signal? Is this signal the same as the original signal f(x)? Discuss why or why not.

2. Image processing
25 points

Describe the effect of each of the following filters. In addition, indicate which filter will cause the most blurring and which, when convolved with a solid (positive) intensity image, will produce the brightest image and which will produce the darkest image. Justify your answers.

0.1	0.1	0.1
0.1	0.1	0.1
0.1	0.1	0.1

0	0	1
0	-2	0
1	0	0

0	0.2	0
0.2	0.4	0.2
0	0.2	0

0	-1	0
0	3	0
0	-1	0

3. Affine Transformations
35 points

$$A=\begin{bmatrix}1&0&0&6\\0&1&0&3\\0&0&1&2\\0&0&0&1\end{bmatrix}\quad B=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&1\end{bmatrix}\quad C=\begin{bmatrix}0&-1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$D=\begin{bmatrix}2&0&0&0\\0&2&0&0\\0&0&2&0\\0&0&0&1\end{bmatrix}\quad E=\begin{bmatrix}2&0&0&0\\0&3&0&0\\0&0&2&0\\0&0&0&1\end{bmatrix}\quad F=\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$G=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}\quad H=\begin{bmatrix}1&0&-1&0\\0&1&0&0\\1&0&1&0\\0&0&0&1\end{bmatrix}\quad I=\begin{bmatrix}1&0&0&1\\0&.6&.8&0\\0&-.8&.6&0\\0&0&0&1\end{bmatrix}$$

a) As discussed in class, any three-dimensional affine transformation can be represented with a 4x4 matrix. Match each of the matrices above to exactly one of the following transformations (not all blanks will be filled):

____ Differential (Non-Uniform) Scaling

____ Reflection

____ Rotation about the z-axis with non-uniform scaling

____ Rotation about the y-axis with non-uniform scaling

____ Translation

____ Rotation about the x-axis

____ Rotation about the y-axis

____ Rotation about the z-axis

____ Shearing along *z* with respect to the x-y plane (z=0 plane unchanged by shear)

____ Shearing along *x* with respect to the y-z plane (x=0 plane unchanged by shear)

____ Rotation about the x-axis and translation

____ Uniform scaling

____ Reflection with uniform scaling

Problem 3 (cont'd.)

b) Consider a line that passes through a point $\mathbf{p} = (p_x, p_y, p_z)$ in the direction $\mathbf{v} = (\cos \alpha, 0, \sin \alpha)$. Write out the product of matrices that would perform a rotation by θ about this line. You should **not** multiply these matrices out, but you do need to write out all of the elements in these matrices.